

ASSIGNMENT 9.

Question 1.

solution

X ~ difference in weight loss

$$X \sim N(\mu, \sigma^2 = (10)^2)$$

$$E(\bar{X}) = \frac{E(X_1 + X_2 + \dots + X_n)}{n} = \frac{n E(X_i)}{n} = \mu$$

$$\text{var}(\bar{X}) = \frac{\text{var}(X_1 + X_2 + \dots + X_n)}{n^2} = \frac{\text{var}(X_i)}{n} = \frac{10^2}{n}$$

$$\Rightarrow \bar{X} \sim N\left(\mu, \sigma^2 = \frac{(10)^2}{n}\right)$$

For 95% C.I, we know:

$$P\left(-1.96 < \frac{\bar{x} - \mu}{10/\sqrt{n}} < 1.96\right)$$

$$E = E_{\text{var}} = 1.96 \times \frac{10}{\sqrt{n}}$$

$$\text{width} = 6 = (\bar{x} + E) - (\bar{x} - E) = 2E$$

$$\therefore E = 3$$

$$\Rightarrow E = \frac{1.96 \times 10}{\sqrt{n}} \Rightarrow 3 = \frac{1.96 \times 10}{\sqrt{n}}$$

$$\Rightarrow 3\sqrt{n} = 19.6$$

$$n \approx 42.6844$$

$$\approx 43$$

Thus

$$\boxed{n \approx 43}$$

Question 2

Solution

The rule is that: If p -value is greater than the level of significance (α), then we fail to reject the null hypothesis (H_0). That is, when the p -value is high we fail to conclude that there are significant differences. However, that does not imply that all means are the same.

From the above statements:

- (a) is clearly wrong according to line above
- (b) is correct
- (c) No, it ~~may~~ may be large, but our sample could not prove that.
- (d) True, when sample size increases, the power of test increases.

Therefore: The correct options are:

b and d

Question 3

(a) Fill in the missing values indicated by a question mark:

Solution:

$$\begin{aligned} \text{(i) mean sq. Residual} &= \frac{\text{mean sq. treatment}}{F\text{-value}} \\ &= \frac{0.8804}{20.31} = 0.0433 \end{aligned}$$

$$\begin{aligned} \text{(ii) Sum sq. Residual} &= \text{mean sq. Residual} \times d.f \\ &= 0.0433 \times 80 \\ &= 3.4678 \end{aligned}$$

(b) How many measurements are there in the study:

Solution:

$$\text{Here, total df} = 80 + 3 = 83$$

Hence, The total number of measurements

$$= 83 + 1$$

$$= \underline{\underline{84}}$$

Question 4

Question 4.

(9) Solution -

null and alternative hypothesis

$$H_0: \mu_1 = \mu_2 = \mu_3$$

H_1 : At least one μ is different than the rest

Let the number of treatment groups be $k = 3$

no. of observation $n = 6 + 5 + 5 = 16$

df. of numerator $= k - 1 = (3 - 1) = 2$

df. of denominator $= n - k = (16 - 3) = 13$

let level of significance $\alpha = 0.05$

$F_{critical} = F_{0.05} \approx 3.8055 \approx 3.81$ from EXCEL function

$$[= F.INV(0.05, 2, 13)]$$

Decision Rule: Reject H_0 if $F > F_\alpha$

To find F value

$$\bar{x} = \frac{6(8.9) + 5(6.08) + 5(6.12)}{6 + 5 + 5} = 7.15 \quad (\text{grand mean})$$

Sum of squares treatment

$$SST = \sum n_i (\bar{x}_i - \bar{x})^2 = [6(8.9 - 7.15)^2 + 5(6.08 - 7.15)^2 + 5(6.12 - 7.15)^2] \\ = 29.404$$

Sum of squares error

$$SSE = \sum (n_i - 1) s_i^2 = (6 - 1)(0.53)^2 + (5 - 1)(0.62)^2 + (5 - 1)(0.47)^2 \\ = 3.816$$

Total sum squares, $SS = SST + SSE = (29.404 + 3.816)$
 $= 33.22$

mean square due to error;

$$MSE = \frac{SSE}{n-k} = \frac{3.816}{13} = 0.293$$

mean square due to treatments

$$MST = \frac{SST}{k-1} = \frac{29.404}{2} = 14.702$$

$$F_{\text{value}} (\text{test statistic}) = \frac{MST}{MSE} = \frac{14.702}{0.293} = 50.17$$

The test statistic value $F = 50.17$ is greater than critical value of $F_{\alpha} = 3.81$

Hence, we REJECT THE NULL HYPOTHESIS

(b) Tukey - Kramer test:

One way Anova test:

Source of variation	SS	df	MS	F
Between groups (treatments)	29.404	2	14.702	50.71
Within groups (Error)	3.816	13	0.293	
Total	33.22	15		

Group means are significantly different.

Continuation (4.6)

Turkey - Kramer Procedure ($q_\alpha = 3.81$, $MSE = 0.293$)

Comparison	Absolute difference	Critical range	Results
ISA to ISP	2.82	0.87	Significantly different
ISP to MSP	0.04	0.92	No significant difference
ISA to MSP	2.78	0.87	Significantly different

$$\text{Critical range} = q_\alpha \sqrt{\frac{MSE}{2} \left(\frac{1}{n_i} + \frac{1}{n_j} \right)}$$

ISA = Island, Squirrels absent, ISP = Island, Squirrels present.

MSP = Mainland, squirrels present

Conclusion

So, group ISA compared to ISP and MSP are significantly different.

